The Influence of Exchange Rate Fluctuations under Capital Control

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Abstract
The research of stabilization under the situation of fixed exchange rate rule and delayed fiscal adjustment has attracted much attention. In this paper, we use ordinary differential equations qualitative theory to discuss the consumption of tradable goods and the exchange rate between the two-dimensional space composed of the equilibrium point, further, to analysis the impact of exchange rate volatility on trade.

Keywords: capital control, stability, saddle

1. Introduction

Usually, an anti-inflation program is the implementation of an exchange rate rule, without an immediate reduction in the fiscal deficit which would make such a rule sustainable in the long term. While a lot of literature concentrated to give up the expected exchange rate rules, some recent works have been to consider the expected deficit reduction policy, allowing the impact of exchange rate rules to survive[1-6]. Michael a Ellis (1996) discussed stabilization under capital controls and compared the details of the results with those obtained under perfect capital mobility. This seems a natural line of research, both because it completes the analysis of the response to a successful stabilization program, in the program the components of the program proceed in stages, and addresses a far more common case.

We consider the effects of a stabilization program that consists of an exchange rate rule and a subsequent fiscal deficit reduction which allows the exchange rate rule to be maintained. Assuming that capital is completely fixed, for simplicity, the ‘exchange rate rule’ takes the particular form of a fined exchange rate. Michael a Ellis (1996) acquired an interesting result that in the case of a tax based stabilization capital mobility allows Ricardian equivalence to hold. He suggested that the timing at which the fiscal reform (which is essentially a change from borrowing to neutral tax financing) takes place does not influence the adjustment. The result for the private sector is essentially the same as if the reform was implemented contemporaneously with the adoption of the exchange rate anchor. He found that without capital mobility this ‘neutrality’ of the timing of a tax increase does not obtain, since Ricardian equivalence does not hold. He also found that the extent to which the timing of fiscal reforms relying on expenditure reductions influence the adjustment does not depend on whether there is capital mobility. However, Michael a Ellis did not fully discussed the existence and stabilization of the equilibrium point of the traded goods and real money balances. In this paper, we used ordinary differential equations and solved this problem thoroughly.

2. Model

The model presented in this section is one of a small open economy with traded and non-traded goods. The general price level is given by the price index

\[ P = E^\sigma P_H^{(1-\sigma)} \]  

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Where $E$ is the nominal exchange rate, $P_T$ is the price of the non-traded good, and $\sigma$ is the weight given to the traded good in the price index. From (1), the inflation rate $\pi$ is

$$\pi = \sigma \pi_T + (1 - \sigma) \pi_H$$

(2)

Where $\pi_H$ the rate of non-traded good price increase and $\pi_T$ is both the rate of traded good price increase and the rate of devaluation. The price of the non-traded good is assumed to be perfectly flexible, therefore following the market clearing condition holds at all times.

$$x_H = c_H + g_H$$

(3)

Non-traded good output is fixed and is denoted by $x_H$ ; $c_H$ is private non-traded good consumption, and $g_H$ is government non-traded good consumption.

Individuals are identical, infinitely lived, and have perfect foresight. The representative individual maximizes the present value of utility, which depends on consumption of the two commodities and the real money stock, and which is assumed to be separable in consumption and money. The utility functional then takes the calculation form

$$\int_0^\infty [u(c_T, c_H) + v(m)] e^{-\delta t} dt$$

(4)

where $\delta$ is the constant rate of time preference, $c_T$ is traded good consumption and $m$ denotes real money balances, defined as nominal money balances deflated by the price index (1). We assumed that both $u(\bullet)$ and $v(\bullet)$ are strictly concave and satisfy Inada type conditions.

The individual's flow budget constraint, expressed in terms of the traded good, is

$$x_T + \frac{(x_H + s)}{\varepsilon} = c_T + \frac{(c_H + T)}{\varepsilon} + \varepsilon^{(\sigma-1)} m \pi + \varepsilon^{(\sigma-1)} \cdot m$$

(5)

where $x_T$ is the fixed flow of traded good income, $s$ is the flow of lump sum government transfers, $T$ is the flow of lump sum taxes, and $\varepsilon$ is the relative price of the traded good in terms of the non-traded good. This relative price is defined as the real exchange rate.

Maximization of (4) subject to (5) and the initial level of $m$ yield the following conditions:

$$\varepsilon = u_T(c_T, c_H) / u_H(c_T, c_H)$$

(6)

$$c_T = \left( \frac{-u_T}{u_T} \right) \left( u_T^{-\sigma} u_H^{(\sigma-1)} v_m - (\delta + \pi_T) \right)$$

(7)

Equation (6) determines the real exchange rate. Equation (7) governs private traded good consumption and from this equation the steady state demand for real balances is implicitly defined by

$$v_m(m^*) = (\delta + \pi_T) u_T(c_T^*, c_H^*)^{\sigma} u_H(c_T^*, c_H^*)^{(1-\sigma)}$$

(8)

Where $*$ denotes steady state values. The government budget constraint, expressed in terms of the traded good, is given by

$$g_T + \frac{g_H}{\varepsilon} + \frac{A + s}{\varepsilon} = A r + \varepsilon^{(\sigma-1)} m \pi + \varepsilon^{(\sigma-1)} \cdot m + \frac{T}{\varepsilon}$$

(9)
Where $g_T$ is government traded good consumption, $A$ is the stock of net foreign assets, and $r$ is the exogenous real interest rate on foreign assets. The flow of government transfers is equal to the component of the inflation tax resulting from non-traded good price inflation,

$$s = (1-\sigma)\pi_H e^\sigma m$$

Equation (10), the fixed exchange rate rule $\pi_T = 0$ and (3) imply that (5), and (7) can be rewritten as follows:

$$m = (x_T - c_T)e^{(1-\sigma)} + (g_H - T)e^{-\sigma}$$

$$c_T = \left(\frac{-u_T}{u_T}\right)(u_T^{-\sigma}u_H^{\sigma - 1}v_m - \delta)$$

3. Stability Analysis

In this paper, we discuss more general situation than that in the paper [8]. Denote $c_T = x$, for a more accurate simulation of exchange rate fluctuations, we consider $u(c_T, c_H) = c_T^\alpha c_H^{\beta}$, $v_m = m^{\alpha-1}$, $\epsilon = \alpha x^{n-1} + bx + c$. In this situation, we discuss the stability of the equilibrium point $(c^*_T, m^*)$ according to equation(11) and (12). The $x$-coordinate of the equilibrium point of equation(11) and (12) satisfied with the following algebraic equation:

$$ax^n - ax_T x^{n-1} + bx^2 + (c - bx_T)x + T - g_H - cx_T = 0$$

In the light of intermediate value theorem, We obtain following results about exist of the equilibrium point of equation(11) and (12).

**Lemma 1** If $cx_T > 0$, and $0 < T - g_H < cx_T$, Then there at least exists a real root of algebraic equation (13) in the interval $[0, x_T]$.

**Lemma 2** If $cx_T < 0$, $T - g_H < 0$ and $T - g_H > cx_T$, Then there at least exists a real root of algebraic equation (13) in the interval $[0, x_T]$.

**Lemma 3** If $a > 0$, and $\max\{T - g_H, T - g_H - cx_T\} < 0$, Then there at least exists a real root of algebraic equation (13) in the interval $[x_T, +\infty)$.

**Lemma 4** If $a < 0$, and $\min\{T - g_H, T - g_H - cx_T\} > 0$, Then there at least exists a real root of algebraic equation (13) in the interval $[x_T, +\infty)$.

**Lemma 5** (Hurwitz Theorem) The necessary and sufficient condition for all the roots of real coefficients of following nth algebraic equations have negative real part

$$\lambda^n + P_1\lambda^{n-1} + \cdots + P_{n-1}\lambda + P_n = 0$$

is that the following Hurwitz determinant:

$$\left| \begin{array}{cccc}
\alpha & \beta & \gamma & \delta \\
\epsilon & \zeta & \eta & \xi \\
\nu & \omega & \theta & \upsilon \\
\phi & \chi & \psi & \rho \\
\end{array} \right| = 0$$
\[ \Delta_1 = \frac{b}{a} - x_T, \Delta_2 = \begin{bmatrix} P_1 & P_3 \\ P_0 & P_2 \end{bmatrix}, \Delta_3 = \begin{bmatrix} P_1 & P_3 & P_5 \\ P_0 & P_2 & P_4 \end{bmatrix}, \ldots, \Delta_n = \begin{bmatrix} P_1 & P_3 & \cdots & P_{2n-1} \\ P_0 & P_2 & \cdots & P_{2n-2} \end{bmatrix} = P_n \Lambda_{n-1} \]

are not less than zero.

Now, we discuss several special cases.

**Case 1 \( n = 3 \)**

In this case, algebraic equation (13) can be rewritten as:

\[
x^3 + \left(\frac{b}{a} - x_T\right)x^2 + \left(\frac{c - bx_T}{a}\right)x + \frac{T - g_H - cx_T}{a} = 0
\]

After simple calculate, Hurwitz determinant become:

\[
\Delta_1 = \frac{b}{a} - x_T, \quad \Delta_2 = \frac{1}{a^2} [(b - ax_T)(c - bx_T) - a(T - g_H - cx_T)],
\]

\[
\Delta_3 = \frac{1}{a^3} (T - g_H - cx_T)[(b - ax_T)(c - bx_T) - a(T - g_H - cx_T)].
\]

According to above lemmas and Hurwitz theorem, it is easy to prove the following theorems

**Theorem1** (i) If \( \frac{b}{a} < x_T < a(T - g_H), b^3 - 4abc + 4a^2(T - g_H) < 0 \), and \( a > 0, b > 0, c < 0 \);

(ii) If \( \frac{b}{a} < x_T < a(T - g_H), b^3 - 4abc + 4a^2(T - g_H) > 0 \), and \( a < 0, b > 0, c < 0 \)

Then the equilibrium point \( (c^*_T, m^*) \) is stable.

**Theorem2** (i) When \( a < 0, b < 0 \), if \( (T - g_H) < \min \{0, cx_T\}, ax_T < b \), and \( c^2 < b(T - g_H) \);

(ii) When \( a > 0, b < 0 \), if \( \frac{b}{a} < x_T < \frac{1}{c} (T - g_H) \), and \( (ax_T^2 + c)^2 < 4x_T(T - g_H) > 0 \).

Then the equilibrium point \( (c^*_T, m^*) \) is stable.

**Case 2 \( n = 4 \)**

In this case, algebraic equation (13) can be rewritten as:

\[
x^4 - x_T x^3 + \frac{b}{a} x^2 + \left(\frac{c - bx_T}{a}\right)x + \frac{T - g_H - cx_T}{a} = 0
\]

At this moment, Hurwitz determinant become:

\[
\Delta_1 = -x_T < 0, \Delta_2 = -\frac{c}{a} < 0, \Delta_3 = -\frac{c^2 - (x_T^2 + b)x_T c + x_T^2(T - g_H)}{a^2} < 0,
\]

...
\[ \Delta_4 = -\frac{T - g_H - cx_T}{a} \Delta_3 < 0 \]

They are not less than zero. According to above lemmas and Hurwitz theorem, it is easy to prove the following theorems

**Theorem 3**

If (i) \( x_T > 0 \); (ii) \( c > 0 \); (iii) \( T - g_H > \max\{ cx_T, \frac{1}{4}(x_T^2 + b)^2 \} \). Then the equilibrium point \((c_T^*, m^*)\) is stable.

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**Case 3 \( n = 5 \)**

In this case, algebraic equation (13) can be rewritten as:

\[
x^5 - x_Tx^4 + \frac{b}{a}x^2 + \left(\frac{c - bx_T}{a}\right)x + \frac{T - g_H - cx_T}{a} = 0 \quad (16)
\]

After simple calculate, Hurwitz determinant become:

\[
\Delta_1 = -x_T < 0, \quad \Delta_2 = -\frac{b}{a} < 0, \quad \Delta_3 = \frac{x_T}{a}[x_T(c - bx_T) - (T - g_H - cx_T)] - \left(\frac{b}{a}\right)^2 < 0, \quad \Delta_4 = -\frac{T - g_H - cx_T}{a}(-\frac{b}{a}) + \frac{c - bx_T}{a}\Delta_3 < 0
\]

They are not less than zero. According to above lemmas and Hurwitz theorem, it is easy to prove the following theorems

**Theorem 4**

If (i) \( x_T > 0 \); (ii) \( \frac{b}{a} > 0 \); (iii) \( \frac{x_T}{a}[x_T(c - bx_T) - (T - g_H - cx_T)] - \left(\frac{b}{a}\right)^2 < 0 \); (iv) \( \frac{c - bx_T}{a}\Delta_3 < \frac{b(T - g_H - cx_T)}{a^2} \).

Then the equilibrium point \((c_T^*, m^*)\) is stable.

**Case 4 \( n = 6 \)**

In this case, algebraic equation (13) can be rewritten as:

\[
x^6 - x_Tx^5 + \frac{b}{a}x^2 + \left(\frac{c - bx_T}{a}\right)x + \frac{T - g_H - cx_T}{a} = 0 \quad (17)
\]
At this moment, Hurwitz determinant become:

$$\Delta_1 = -x_T < 0, \Delta_2 = 0, \Delta_3 = -\frac{c}{a}x_T < 0, \Delta_4 = \left(\frac{c}{a}\right)^2 < 0$$

$$\Delta_5 = -\frac{1}{a^2}[(T - g_H - cx_T)^2 x_T^3 + \frac{c^2}{a}(c - bx_T)] < 0, \Delta_6 = \frac{T - g_H - cx_T}{a} \Delta_5 < 0$$

They are not less than zero. According to above lemmas and Hurwitz theorem, it is easy to prove the following theorems

**Theorem 5**

If (i) $x_T > 0$; (ii) $\frac{c}{a} > 0$;

$$(ii) \frac{b(T - g_H - cx_T)}{a} > 0;$$

$$(iii) (T - g_H - cx_T)^2 x_T^3 + \frac{c^2}{a}(c - bx_T) > 0.$$  

Then the equilibrium point $(c_T^*, m_T^*)$ is semi-stable.

**Case 5 $n = 7$**

In this case, algebraic equation (13) can be rewritten as:

$$x^7 - x_T x^6 + \frac{b}{a} x^2 + \left(\frac{c - bx_T}{a}\right)x + \frac{T - g_H - cx_T}{a} = 0$$  \hspace{1cm} (18)

At this case, Hurwitz determinant become:

$$\Delta_1 = -x_T < 0, \Delta_2 = 0, \Delta_3 = -\frac{b}{a} < 0, \Delta_4 = \left(\frac{b}{a}\right)^2 < 0$$

$$\Delta_5 = -\frac{b^3}{a^3} \frac{x_T}{a^2} \left[-(c - bx_T) x_T^2 + \frac{b}{a} x_T (T - g_H - cx_T)\right] < 0$$

$$\Delta_6 = -\frac{c - bx_T}{a} \Delta_5^2 + \frac{T - g_H - bx_T}{a} D < 0$$

$$\Delta_7 = \frac{T - g_H - cx_T}{a} \Delta_6 < 0.$$  

Where $D = \frac{(c - bx_T) (T - g_H - cx_T)}{a^2} (1 + x_T) + \frac{(T - g_H - cx_T)^2}{a^2} - \frac{b(b - cx_T)^2}{a^2} x_T^2$

They are not less than zero. According to above lemmas and Hurwitz theorem, it is easy to prove the following theorems
Theorem 6

If (i) $x_T > 0$; (ii) $\frac{b}{a} > 0$;

(ii) $\frac{c-bx_T}{a} > 0$;

(iii) $\frac{T-g_H-cx_T}{a^2} > 0$;

(iv) $abx_T^3 - bcx_T + (b-ax_T)(T-g_H) > 0$;

(v) $D < 0$

Then the equilibrium point $(c_T^*, m^*)$ is semi-stable.

4. Conclusions

Michael (1996) exchange rate movements on the paper is a decreasing function of tradable goods, this conclusion is based on tradable goods are normal goods under this assumption. In comparison, the authors Stability of Michael Model under Capital Control an article relaxes this assumption, the exchange rate movements in either direction, can be non-tradable goods normal goods. On this basis, we have come to a variety of economic and corresponding steady-state stability condition, and discussed the stability of each type of equilibrium. According to the analysis, we get both the developed and developing countries by the body can reach their equilibrium, and in a periodic steady state. The stability of this equilibrium only with the consumption of tradable goods in the utility function of the weight and the consumption of tradable goods, whereas the consumption of tradable goods affecting factors in this paper the performance of tradable goods income flow. In specific cases, by controlling the consumption of tradable goods and tradable goods revenue inflows can make economic equilibrium and stability of the state. Meanwhile, we also found that the steady state at a low level and a high level of steady state exists between an unstable region, that is in the process of economic development, there will be a period of economic imbalances, the economic crisis. At this time, if through appropriate adjustments again reaches equilibrium when the economy will be in a high level of stability, on the contrary, if the adjustment properly, the economy will return to a lower level of stability. For developing countries, this is both an opportunity and a challenge.

Based on the above analysis, we further assumed that the exchange rates fluctuate freely, obtained in conditions of capital controls, exchange rate fluctuations will lead to government deficit and current account trade deficit.

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References


